

* Fit a straight line to the data:

(i)	x	1	5	6	7	8	9
	y	4	7	8	12	10	15

Find value of y if $x = 10$

Solution! x y xy x^2

1	4	4	1
5	7	35	25
6	8	48	36
7	12	84	49
8	10	80	64
9	15	135	81
<u>36</u>	<u>56</u>	<u>386</u>	<u>256</u>

Here $n = 6$,

$$\sum x = 36, \sum y = 56$$

$$\sum xy = 386, \sum x^2 = 256$$

$$\text{Suppose } y = a + bx \quad (\star)$$

Corresponding normal equations are,

$$\sum y = na + b \sum x \quad (i)$$

$$\sum xy = a \sum x + b \sum x^2 \quad (ii)$$

$$(i) \Rightarrow 56 = 6a + 36b \quad (1)$$

$$386 = 36a + 256b \quad (2)$$

$$(1) \times 6 \Rightarrow 560 = 36a + 216b \quad (3)$$

$$(2) - (3) \Rightarrow -$$

$$-180 = 40b$$

$$b = 4.5$$

$$\text{Put } b = 4.5 \text{ in } (1)$$

$$(1) \Rightarrow 56 = 6a + 36 \times 4.5$$

$$6a = -106$$

$$a = -17.67$$

$$\text{by } (\star), \quad y = -17.67 + 4.5x$$

$$\text{Put } x = 10$$

$$y = 27.33$$

* Fit a straight line by least square method

- (ii) $\begin{array}{cccccc} x & 1 & 2 & 3 & 4 & 5 \\ y & 4 & 9 & 15 & 22 & 30 \end{array}$
Find value of y if $x=6$.
- (iii) $\begin{array}{ccccc} x & 0 & 1 & 2 & 3 & 4 \\ y & 1 & 1.8 & 2.3 & 4.5 & 6.3 \end{array}$
Find value of y if $x=5$.
- (iv) $\begin{array}{ccccccccc} x & 1 & 2 & 4 & 5 & 6 & 8 & 9 \\ y & 2 & 5 & 7 & 10 & 12 & 15 & 19 \end{array}$

* Fit a parabolic curve or second degree curve to the data:

(i)	Year	1961	1962	1963	1964	1965	1966	1967
	Sales	80	90	92	83	94	99	92

Solution! Let $y = a + bx + cx^2$ — *)

Corresponding normal equns are,

$$\sum y = na + b \sum x + c \sum x^2 \quad \text{--- (i)}$$

$$\sum xy = a \sum x + b \sum x^2 + c \sum x^3 \quad \text{--- (ii)}$$

$$\sum x^2 y = a \sum x^2 + b \sum x^3 + c \sum x^4 \quad \text{--- (iii)}$$

Year	Sales	$x = t - 1964$	x^2	x^3	x^4	xy	x^2y
t	y						
1961	80	-3	9	-27	81	-240	720
1962	90	-2	4	-8	16	-180	360
1963	92	-1	1	-1	1	-92	92
1964	83	0	0	0	0	0	0
1965	94	1	1	1	1	94	94
1966	99	2	4	8	16	198	396
1967	92	3	9	27	81	276	828
	630	0	28	0	196	56	2490

$$(i) \Rightarrow 630 = 7a + bx0 + 28c \quad (\text{ie}) \quad 7a + 28c = 630 \quad \text{--- (1)}$$

$$(ii) \Rightarrow 56 = ax0 + bx28 + cx0 \quad (\text{ie}) \quad 28b = 56 \Rightarrow b = 2$$

$$(iii) \Rightarrow 2490 = 28a + bx0 + 196c \quad (\text{ie}) \quad 28a + 196c = 2490 \quad \text{--- (2)}$$

$$(1) \times 4 \Rightarrow 28a + 112c = 2520 \quad \text{--- (3)}$$

$$28a + 196c = 2490 \quad \text{--- (2)}$$

— — —

$$(3) - (2) \Rightarrow -84c = 30 \Rightarrow c = -0.357$$

$$\therefore (1) \Rightarrow 7a + 28x(-0.357) = 630 \Rightarrow a = 91.428$$

$$(*) \Rightarrow y = 91.428 + 2x - 0.357x^2$$

* Fit a parabolic curve or second degree curve

(ii)	Year	1929	1930	1931	1932	1933	1934	1935	1936	1937
	Profit	52	56	57	58	60	61	61	60	59
(iii)	Year	1991	1992	1993	1994	1995	1996	1997		
	Production	9	11	15	16	19	21	23		
(iv)	Year	1996	1997	1998	1999	2000	2001			
	Sales	24	27	26	29	31	35			
(v)	Year	2011	2012	2013	2014	2015	2016			
	Profit	31	33	37	39	42	46			

* Fit an exponential curve of the form $y = ab^x$:

x	0	1	2	3	4
y	1.5	2.5	3.5	5	7.5

Solution! Let $y = ab^x$ — (i)

taking log on both sides,

$$\ln y = \ln(ab^x)$$

$$= \ln a + x \ln b$$

Let $\ln y = Y$, $\ln a = A$ and $\ln b = B$

$$Y = A + xB \quad \text{— (ii)}$$

Corresponding normal equns are,

$$\sum Y = nA + B \sum x \quad \text{— (iii)}$$

$$\sum xY = A \sum x + B \sum x^2 \quad \text{— (iv)}$$

x	y	$Y = \ln y$	xY	x^2
0	1.5	0.405	0	0
1	2.5	0.916	0.916	1
2	3.5	1.253	2.506	4
3	5	1.609	4.827	9
4	7.5	2.015	8.06	16
10		6.198	16.309	30

$$6.198 = 5A + 10B \quad \text{— (1)}$$

$$16.309 = 10A + 30B \quad \text{— (2)}$$

$$(1) \times 2 \Rightarrow 12.396 = 10A + 20B \quad \text{— (3)}$$

$$(5) - (3) \Rightarrow \underline{\underline{3.913 = 10B}}$$

$$B = 0.3913 \Rightarrow b = e^B = 1.4789$$

$$(1) \Rightarrow 5A + 10 \times 0.3913 = 6.198$$

$$5A = 2.285$$

$$A = 0.457 \Rightarrow a = e^A = 1.5793$$

$$\therefore y = 1.5793 \times 1.4789^x.$$